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Niveaux de référence pour l'enseignement des mathématiques en Europe

Reference levels in School Mathematics Education in Europe

National Presentation

SPAIN

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1. General description of the school system

The Spanish Parliament by virtue of the law called LOGSE (1990) fixed the structure of the compulsory educational system (age 6 to 16) as follows:

- **Primary Education** (***Educación Primaria***). The 6 years are divided into three two-year cycles corresponding to ages 6 to 8, 8 to 10 and 11-12.

- **Secondary Education** (***Educación Secundaria Obligatoria***). The 4 years are divided into two two-year cycles, corresponding to ages 12 to 14 and 14 to 16.

The Spanish government through the Ministry of Education and Culture (MEC) fixes the National Curriculum specifying general goals as well as the minimum educational requirements (***enseñanzas mínimas***) for each core subject and the minimum timetable requirements (see §5.1 for regional characteristics).

Each region thus has a fixed curriculum, in accordance with the National Curriculum, and then each School is supposed to develop its own project (***Proyecto Curricular de Centro***) according to which each teacher can fix his/her classroom program. Note that the time distribution of contents may vary from one place to another.

In compulsory education, assessment in each subject is continuous and integrated and is carried out collectively by the team of teachers responsible for a given group of students, with the coordination of the form master and advice from the Guidance Department (***Departamento de Orientación***).

Pupils who have attained the required objectives at the end of their compulsory education, receive the certificate of secondary education (***Graduado en Educación Secundaria***). This qualification enables students to gain access either to the ***Bachillerato*** (2 years) or to the ***Modulos Profesionales*** from which they may continue to higher level vocational studies or university studies (after passing a university entrance examination - PAAU).

Those pupils between the ages of 16 and 21 who do not reach the objectives of Compulsory Secondary Education and consequently do not have the Certificate of Secondary Education, have the possibility of enrolling in special social courses (***Cursos de Garantía Social***) of 1 or 2 years, related to initial vocational training, specific workshops, general training for working, courses for people with special needs, etc. Such courses are organised by various centres and institutions that may give a certificate of aptitude (***Certificado de Competencia***, see http://www.mec.es/educacion/siseduc.html).

In Primary Education, pupils study mathematics 4 hours per week for 6 years. In Secondary Education, pupils study mathematics 3 hours per week, at a rate of 35 weeks per year for 4 years. Sometimes during the second cycle, certain schools add 1 hour of mathematics per week thanks to optional classes (***créditos variables***). One hour of class lasts 50 min. In some regions of Spain the last year of Secondary Education is offered in two options (A and B), which have equivalent content but different terminal objectives: while option A is for weaker students leaving the school system after this year, option B includes more formal considerations and is for students trying to pursue their non-compulsory studies.

Note that there is neither a global exam in the schools nor a national test or some external form of assessment, thus the ***Graduado en Educación***

Secundaria is given to students by their school after all subjects have been passed.

2. Principal Mathematics Objectives

For the proposed National Curriculum of mathematics (12-16) there is a decalogue of basic general objectives (MEC, 1992) to be achieved by the end of this compulsory period, which can be summarised as follows:

a) To incorporate the language and arguments typical to mathematical reasoning (numerical, graphical, geometrical, logical, algebraic, probabilistic) in order to be able to communicate in a precise and rigorous way.

b) To use logical ways of thinking, to formulate and verify conjectures, to make inferences and deductions, and to organise and relate information related to daily life and problem solving.

c) To quantify real situations by virtue of data, measures and numbers making the appropriate computations.

d) To elaborate personal strategies for analysing concrete situations and for problem solving using various resources and instruments, analysing the final results obtained.

e) To use elementary techniques for collecting data to obtain information about phenomena and situations and to represent this information by means of numerical or graphical forms in order to evaluate it.

f) To recognise the diversity of realities and to see how different points of view can be used to explain it (deterministic/random, finite/infinite, exact/approximate).

g) To identify shapes and spatial relations present in reality, analysing geometrical properties and relationships.

h) To identify the mathematical elements (statistical data, graphics, maps, computations, etc.) which are present in news, pools, publicity etc., critically analysing their functions and possibilities for a better understanding of messages.

i) To approach day to day situations and problem solving in a mathematical way (exploring alternatives, using a precise language, etc.).

j) To know and value personal mathematical skills in order to deal with situations that require their use or in which one can enjoy the creative, manipulative, aesthetic or useful aspects of mathematics.

So, clearly the official curriculum has quite acceptable general objectives, which are then specified in terms of skills and content (see §3), and precise terminal aims are formulated. Thus the intended curriculum in Spain combines in a reasonable way the mathematical world, the general abilities to be acquired by students and the applications of mathematics.

Concerning the implemented curriculum, and the strong influence that textbooks have in teaching, one can observe the tendency to devote more attention to traditional content (numbers, operations, algebraic manipulations, exercise solving, etc.) than to working on skills and applications. Also some topics like geometry, probability and statistics that are well oriented in the curriculum (see §3) are either being overlooked or converted into a formulaic game.

3. Basic content

The basic-minimal official curriculum is quite demanding and very specific. Official publications have been extremely well prepared with details concerning general objectives, content (facts, skills, attitudes) and criteria for assessment.

Additional documents giving examples of developments, alternative sequentiations, didactical orientations, resources to be used, etc. have been published and distributed. The following list is a brief résumé of the main concepts and skills of the official syllabi for Secondary School Education (12-16):

1. Numbers and operations: meanings, strategies and symbolisation

Main concepts: Natural numbers. Integers. Decimals. Fractions; percentages; numerical meanings and numerical uses; notations and operations; numerical relations (ordering); proportional magnitudes; factors, approximation and estimation of quantities; basic operational algorithms; use of calculators and other devices; algebraic language.

Main skills: use of appropriate numbers; number sense; numerical representations; mental calculus; use of algorithms for all basic operations, use of different strategies; application to real life problems; problem solving, ...

2. Measures, estimation and calculus of magnitudes

Main concepts: Measurement of magnitudes. Units. Systems of measurement. The metric system. Time measurement. Angle measurement. Approximate measurements. Indirect measures. Trigonometrical ratios and their relation ($\sin\theta$, $\cos\theta$, $\tan\theta$). Instruments for measurement.

Main skills: Naming, expressing and representing measures; use of formulas for lengths, areas and volume; use of instruments; use of trigonometric ratios; bounded errors arising in measurements; applying measures.

3. Representation and organisation in the space

Main concepts: Geometrical elements in the plane and in space (points, lines, planes; parallelism, perpendicularly, incidence, ...) Cartesian

references in the plane and in space; spherical coordinates; figures and shapes (classification; elements and characteristic of basic figures in the planes and in space); similar figures; scales in representations (maps, models, ...); proportionality and the Tales theorem; isometric transformations in the plane (translations, rotations, symmetries).

Main skills: language applied to geometrical configurations, use of references; making geometrical models; making elementary representations; identification of similar shapes; use of Tales theorem; solving geometrical problems; classification; study of figures; establishing relations; ...

4. Interpretation, representation and analysis of information

Main concepts: (A) Information on deterministic phenomena: functional dependence, tables, graphs, formulas; characteristics of graphs; elementary functions (linear, quadratic, exponential,). (B) Information on random phenomena: samples; frequencies; percentages; statistical graphics; statistical parameters; dependency between variables.

Main skills: use of alternative representations; selecting sources; selecting samples; avoiding false relations; dealing with graphs; studying data; ...

5. Treatment of chance

Main concepts: random phenomena, events; experiments; frequencies; probabilities; Laplace's rule; conditional probability; dependence and independence; ...

Main skills: describing random phenomena, reading and producing graphics; assigning probabilities; making diagrams and tables for compound probabilities; avoiding misunderstandings concerning chance; ...

Even if the official curriculum is expressed in terms of basic-minimal, it is usual to treat the specific objectives as maximum objectives. This

phenomenon is accentuated by the generalised use of textbooks for which the authorities fix not only minimums to be satisfied, but also maximums.

In all cases ***attitudes*** to promote are also specified for each block. They are concerned, mainly, with working mathematically in an organised form, the appreciation of mathematical values and the spirit of research.

Note again that in some regions all contents can be distributed in Secondary Education, following different patterns (grouping statistics, distributing geometry, etc.) but in some regions a cycle distribution has been fixed.

4. Sample topics

4.1 Quadratic equations

This subject belongs to the second cycle of Secondary Education and is generally taught in the last year. The quadratic function (with its analytical and graphical representation) is treated before the general equation and the parabola. Sometimes, “completing the square” is presented, but usually the well-known roots formulae are systematically applied. In some regions of Spain, where both options A and B are available for the second cycle, quadratic equations are placed in option B.

Habitually, too much time is dedicated to the mechanical resolution of this equation rather than to the meaning of its writings. No time or little time is spent on the relations between the factorisation form $a(x - x_1)(x - x_2)$, the standard canonical form $\beta + k(x - \alpha)^2$ and their graphical or geometric senses.

4.2 Pythagorean theorem

This theorem belongs to the intended curriculum and it may appear either in the first or in the second cycle. Pythagoras’ equality and the theorem itself are not distinguished. This equality is introduced as a relation between the areas of the square of right-angled triangle lengths, and it is used to calculate the measurement of an unknown. In fact, even if the geometrical approach precedes the arithmetic approach, its application remains basically algebraic.

Almost no attention is paid to equivalence between Pythagoras' equality and the orthogonality of the sides, i.e. this theorem does not intervene to justify a possible step of deduction. Even when one presents other geometrical properties of the right-angled triangle, they are employed to algebraic ends.

The easiest visual proof is often given (adapted from the famous ***Chou pei suan ching*** - Chinese proof) and it is used for some geometrical applications (making a layout with tangram pieces or studying procedures of cutting). Sometimes, Pythagorean theorems are introduced from a historical perspective (e.g. the knots determining the 3-4-5 triangle, Pythagoras' life, geometrical construction of squares roots, Pythagorean triplets, ...).

4.3 Similarity

While proportionality and scales are already present at the end of primary school, geometric similarity of plane shapes appears in the intended curriculum at the end of the second cycle. This includes Thales theorem, its corollaries and their applications. Sometimes, homotheties are introduced to verify the similarity between figures (subordinating the geometrical idea of transformation), and the trigonometrical ratios are defined from a family of similar right-angled triangles. It is necessary to note that, surprisingly, similarity for three-dimensional shapes is not touched on but relations between areas and volumes of similar figures is included.

Similarity is always a difficult topic with misunderstandings in representations of similar figures when they are presented in a perspective picture. The comprehension and application of the k-k^2-k^3 property (enlargement/reduction coefficient between lengths, areas and volumes) for effective problem solving continue to be a difficulty.

4.4 Word problems

Word problems, in the general framework of problem solving, are always present at the end of Primary Education and throughout Secondary Education. Dealing with these problems is a constant difficulty because they require

careful thinking and some process of modelling, which is beyond the usual routine exercises. In addition, working in context is not usual, so some obstruction may arise from the "context" conditions (locating the knowns and unknowns, giving interpretations of results, excluding unacceptable solutions, etc.). The large quantity of fictional and ad-hoc situations being presented in textbooks as "real contexts" is remarkable. Few problems encourage the real challenge, the exploration of additional concepts and the quest for new methods (see §5.5).

4.5 Percentages

At the end of Primary Education particular cases of percentages (10%, 50%, ...) have already been presented. In Secondary Education percentages are introduced first as fractions and later they become identified as particular cases of proportionality. Formulas of interest are given and, in theory, relations with statistical data and probabilities, frequencies, etc., can be presented.

Needless to say, students have difficulties in making the correct percentages if the wording of the problems is not clear (while mechanical computations become straightforward). The distinction between the multiplying coefficient and the increasing/decreasing rate is not often approached.

4.6 An additional topic: history of mathematics

Elements of the History of Mathematics are not included in the basic intended curriculum but, nevertheless, it is a common practice of teachers and a traditional part of textbooks always to include some historical references either as a motivational tool (numerical representations, instruments, ...) or as biographical items (Pythagoras, Thales, Euclides, Fibonacci, Pascal, Descartes, ...) or as examples of applications (castle heights, tax paying, maps, ...). It is surprising what a small amount of historical data of the XXth century is included. The existence of Spanish comics about the history of mathematics, some popular recreational books, the experience of some radio programs (Huelva) dramatising the history of mathematics, the exhibits on

traditional measures, and the interest of many teachers in historical aspects have contributed to the presence of this topic in the classrooms. Nevertheless, training in the mathematical elements of culture is practically never evaluated.

5. Further considerations

5.1 Regional characteristics

As mentioned above, the Parliament framed the LOGSE law to be applicable to all parts of Spain, and the Ministry of Education and Culture (MEC) fixed the national curriculum specifying basic goals and objectives, minimum content and timetables for each subject. These criteria must be applied to all schools in Spain.

Since Spain has been structured in 17 Autonomous Communities (***Comunidad Autonómica***), authorities in each Community are allowed to develop the regional curriculum taking into account the MEC regulations and constraints. Those Autonomous Communities with their own language (Basque Country, Catatonia, Galicia, and Valencia) are able to devote 10 per cent of the total timetable for language and cultural learning.

While in other subjects the regional curricula may have quite a lot of different items added to the minimal curriculum, in the case of mathematics there are not so many differences, but still one can find different sequential approaches, different applications and examples; etc. This has a consequence for textbooks since they need to be approved in each Autonomous Community.

5.2 Implementation strategies

After the National Curriculum was fixed and the regional curricula were set by the Autonomous Communities, the MEC and the educational authorities of the communities organised many training courses for teachers involved in public education. In the years just after the reform these activities were quite

numerous. They were non compulsory and they had some small implication on salaries (through accumulated points derived from attendance of courses). Many examples of mathematics curriculum developments were published and distributed to public centres by academic authorities.

In many places either Centres for Teachers or Resource Centres have been established but only in some of these centres have trained mathematics coordinators been assigned. While at the beginning of the reform many maths teachers received the training to become mathematics coordinators (formatters), in very well organised courses, including international participation of well-known mathematics educators, most of these teachers went back to their original schools because the demanding job of group work in zones including many centres did not received enough incentivization.

5.3 Teacher training

The requirements for teaching mathematics at the Primary Level are completing the ***Bachillerato*** (in any of its specialities) and the 3-year program (1 cycle) of ***Maestro*** at the School of Education of any University.

To teach mathematics at Secondary Education the requirements are completion of the ***Bachillerato*** and the 4-5 year program (2 cycles) of university studies giving the degree of ***Licenciado***, ***Ingeniero*** or ***Arquitecto***. Theoretically, the ***Licenciados en Matemáticas*** from any Mathematics Faculty are the ideal candidates to take over these jobs. At present a 600 hour course (including 200 hours of Psychological-Pedagogical preparation, 200 hours of practical tutored work in a school and 200 hours of Didactics of Mathematics) called ***Curso de Cualificación Pedagógica*** is compulsory after the university degree in order to teach in any private or public school (a shorter course was compulsory before, but only in the public system).

In service training for teachers is not compulsory and it is mainly based upon activities organised either by the authorities or by the various teachers' associations of mathematics.

Most people in Mathematics Education and its research are teaching in Schools of education and may offer (Barcelona, Granada, Valencia, ...) post-graduate courses, masters and PhD programs on Mathematics Education. This has a positive impact on some teachers who became involved in these programs. Activities of the ***Institutos de Ciencias de la Educación***, which are university institutes, also have training impact.

5.4 Resources available to teachers

Teachers may find in their schools a small library, a computer lab to be used for teaching purposes and audio-visual facilities. In some Communities an official centre (***Programa Informática Educativa***, ...) may supply educational software and on-line interactive resources. A program to place Internet connection in each centre is being developed fast.

In the zones where there is a Resource Centre or a Centre for Teachers, teachers may receive training courses, seminars, information, etc. and may ask for materials to be used in their classroom for short periods of time (videos, games, geometric shapes, books, graphic calculators, ...).

Professional associations offer congresses, courses, seminars, lectures, ... and periodic publications of a didactical nature but they do not facilitate hands-on material resources or paradigmatic prêt-à-porter activities. Any one of the almost 4000 mathematics teachers associated with one of the local societies receives the journal Suma, published by the Federation of all mathematical societies of teachers plus the concrete Bulletin, Newsletter or Journal of their own association.

5.5 Problems and improvements already detected

The reform of compulsory education has been completed so all students are now following the new system.

Some “problems” detected in relation to mathematics education during the compulsory years in the new development are the following (the ordering is random):

- Many teachers in Primary Education are not confident of their own knowledge of mathematics, or their training was not appropriate (any ***maestro*** can teach mathematics independent of his/her speciality - Music, Physical Education, Foreign Language, Special Education, and Primary Education).

- Many teachers in Secondary Education are teaching mathematics but had degrees in Science, Engineering, etc., so in some communities there is a lack of mathematicians. Training of teachers may need further efforts.

- Publishers have edited textbooks not according to the ideal mathematical content, but to the most popular selling factors. Thus most textbooks, while adapted to the legal curricula, present traditional activities (paper and pencil exercises, limited use of technology, algorithmic exercises, ...). Moreover, they often do not support the research of mathematical concepts or methods that exceed the minimal aspect of the basic curriculum (see §3).

- Assessment, attention to diversity, gender issues, dynamics of the classroom, etc. are often being considered according to traditional patterns. Mathematics teachers are confused by pupils 12-16 years old who have no interest at all in the subject or in their own education.

- Social views of mathematics are quite negative, respect for teachers is lower now, and the lack of motivation of pupils for learning (as well as lack of correct behaviour) may affect some classes.

- Pupils having completed their Secondary Education may have less concrete knowledge of mathematical items, but at least in theory, they are better prepared in relation to mathematical skills. The problem of coordination between Primary, Secondary and Post-Secondary education needs further consideration.

- The decreasing demography will close doors to young teachers and active teachers will become older which may affect innovative aspects.

Some of the improvements seen in the last decade are, for example,

- Compulsory education has expanded from the age limit of 14 to 16, i.e., many more people will receive more intensive mathematics education. Since the job market starts legally at 16 years of age, the bizarre situation of “leaving school at 14 and not working till 16” has disappeared.

- In (almost) every Autonomous Community there is a Society or Association of Mathematics teachers and there is a Federation of all such bodies (see appendix). This did not exist in the past and does not exist in other subjects. Activities of these associations have greatly helped the associates to be involved in mathematics education activities.

- The national mathematics curriculum was well designed so its educational goals are, in principle, acceptable and adequate. The administration made quite an economical effort to make this reform possible.

- The decreasing demography in Spain is closing schools and classrooms so in the public system the number of students per classroom has gone down (25 is now quite frequent) and public educators have more colleagues to help in remedial activities.

- The growing number of mathematics educators and the research done by them has a positive influence on the general education system.

5.6 Data on general/local results

The evaluation of Secondary Education in Spain is far from being completed. Currently there are some initiatives to make this evaluation possible on a large scale. Since there are no official final exams at the end of Secondary Education it is difficult to present quantitative results. Results coming from TIMSS (where Spain had mediocre results) had no implication at all beyond some quotes in the newspapers.

5.7 Examples of inspiring activities

Let us mention here a few examples of inspiring activities:

- When the authorities succeed in having an active Resource Centre or Centre for Teachers and have a well trained teacher to become mathematics coordinator of the centre's zone, then schools benefit greatly from activities organised for them, weekly group work, regular visits of the coordinator, etc. This would also include the support of ***Programa Informática Educativa***.

- Having ICME-8 in Seville helped a lot to organise teachers' associations and to increase mathematics education activities.

- There are a growing number of participants in the mathematical Olympiads or contests (such as ***Proves Cangúr***), which are being organised for various age groups.

- Some congresses for teachers held at a local level (***Jornadas***) or at a national level (JAEM every 2 years) may involve many teachers (e.g. 800, ...).

- Some special events like math exhibits, the contest of "Mathematics and photography", the radio programs broadcast by the Thales Society, the "Recreational Mathematics Days" organised by M. Pazos in La Coruña, the "Maths-Magnum" organised by D. Barba and L. Segarra in Barcelona, etc. are interesting and celebrated math events.

- The existence of various PhD programs in Mathematics Education (Barcelona, Granada, Valencia, ...) has permitted some teachers to become active in this field of research and to generate other training activities which have their own interest.

- Some TV programs and videos devoted to mathematics generated quite interesting audio-visual materials (A. Perez, R. Perez-Gomez, ...).

- To commemorate the World Mathematical Year 2000, many activities were held in all of Spain (http://dulcinea.uc3m.es/ceamm/). In addition to the specialised activities, it is interesting to note that certain ideas have had immediate repercussions on the general public (the stating of small problems or mathematical properties in public transport; cycles of mathematical cinema; exhibition of old books, photographs and other mathematical objects; the creation of carpets of flowers with the WMY's logo; ...).

References

- Alonso, F. et altri (1988) ***Aportaciones al debate sobre las matemáticas***. Valencia: Mestral.
- Alsina, C. et al (1988) ***Mathematics education in Spain***. Nat Pres. ICME-6, Budapest (ICMI Bulletin).
- Alsina, C., Fortuny, J.M., Giménez, J. (1992) ***Bon dia mates*** (12-14) (14-16), Barcelona: Dep. Ens. Genealitat Catalunya.
- Cero Grupo (1988) ***Matemáticas. De 12 a 16***. Valencia: C.C. Generalitat Valenciana.
- Dep. Ensenyament Generalitat Catalunya (1992) ***Curriculum Educació Secundaria Obligatoria. Primer nivell de concreció***. Barcelona: Pub. Dep. Ens. Gen. Catalunya.
- Gutiérrez, A. et al. (1991): ***Área de Conocimiento Didáctica de la Matemática***. Madrid: Síntesis.
- Kilpatrick, J., Rico, L., Sierra, M. (1992) ***Educación Matemática e Investigación***. Madrid: Síntesis.
- MEC (1989) ***Diseño Curricular Base. Educación Secundaria Obligatoria I, II***. Madrid: MEC.
- MEC (1992) ***Educación Secundaria Obligatoria: Matemáticas***. Madrid; Pub. MEC.
- Negro, A., Benedicto, C., (1993), ***Diseño Curricula para el Área de Matemáticas. ESO***. Madrid: Síntesis.
- Sierra, M. (1999) ***Mathematics Education in Spain.*** Nat. Pres. ICME-8, Seville: Proc. ICME-8 (457-470).

Appendix

Following is the list of the Spanish Societies of Mathematics Teachers:

- ***Federación Española de Sociedades de Profesores de Matemáticas***
- ***Asociación Gallega de Profesores de Educación Matemática AGAPEMA***
- ***Federació d'Entitats per l'Ensenyament de Matemàtiques a Catalunya FEEMCAT***
- ***Organización Española para la Coeducación Matemática ADA BYRON***
- ***Sociedad Andaluza de Educación Matemática THALES***
- ***Sociedad Aragonesa de Profesores de Matemáticas PEDRO SÁNCHEZ CIRUELO***
- ***Sociedad Asturiana de educación matemática AGUSTÍN DE PEDRAYES***
- ***Sociedad Canaria de Profesores de Matemáticas ISAAC NEWTON***
- ***Sociedad Castellano-Leonesa de Profesores de Matemáticas***
- ***Sociedad Castellano-Manchega de Profesores de Matemáticas***
- ***Sociedad de Educación Matemática de la Región de Murcia***
- ***Sociedad de Enseñantes de Ciencia de Galicia ENCIGA***
- ***Sociedad Extremeña de Educación Matemática VENTURA REYES PRÓSPER***
- ***Sociedad Madrileña de Profesores de Matemáticas EMMA CASTELNUOVO***
- ***Sociedad Matemática de Profesores de Cantabria***
- ***Sociedad Melillense de Educación Matemática***
- ***Sociedad Navarra de Profesores de Matemáticas TORNAMIRA***
- ***Sociedad PUIG ADAM de Profesores de Matemáticas***
- ***Sociedad Riojana de Profesores de Matemáticas A PRIMA***
- ***Societat d'Educació Matemàtica de la Comunitat Valenciana AI-KHWARIZMI***